

Precious Metal Futures

Futures contracts are typically valued as a spread to the forwards curves. This spread is the EFP (Exchange of Futures for Physical). This function spec outlines the introduction of this spread as a basis risk factor into system and the transformation of sensitivities with respect to futures contracts to those with respect the spot contract, forward offered rates, and basis.

If we let $f(t,T)$ forward offered curve at T^{th} maturity and $S(t)$ represent the spot rate for the precious metal at time t , then the forward curve at T is defined as:

$$For_s(t, T) = S(t)(1 + f(t, T) * T) \quad (1)$$

The basis is then defined as the difference between corresponding T futures price and forward price calculated in (1), thus:

$$Basis(t, T) = Fut_s(t, T) - For_s(t, T) \quad (2)$$

Using (1), we can then write the definition as:

$$Basis(t, T) = Fut_s(t, T) - S(t)(1 + f(t, T) * T) \quad (3)$$

Currently, for each precious metal, we will get sensitivities with respect to the futures contracts $\overline{\mathbf{Fut}}_S = \{Fut_S(T_1), \dots, Fut_S(T_N)\}$, the spot contract S , and forward offered curve $\bar{\mathbf{f}} = \{f(T_1), \dots, f(T_N)\}$.

Using (2), we see that our new basis risk factor allows the decomposition of a futures contract into:

$$Fut_S(t, T) = For_S(t, T) + Basis(t, T) \quad (4)$$

This decomposition allows the transformation of the sensitivities with respect to $\overline{\mathbf{Fut}}_S = \{Fut_S(T_1), \dots, Fut_S(T_N)\}$ into those with respect to $\bar{\mathbf{f}} = \{f(T_1), \dots, f(T_N)\}$ and $\overline{\mathbf{Basis}} = \{Basis(T_1), \dots, Basis(T_N)\}$.

To derive the transformations we assume that a pricing function $F := F(\overline{\mathbf{Fut}}_S) = F(\{For_S(T_1) + Basis(T_1), \dots, For_S(T_N) + Basis(T_N)\})$ is equivalent to a pricing function $G := G(S, \bar{\mathbf{f}}, \overline{\mathbf{Basis}})$.

This gives the following equations:

$$\frac{\partial G}{\partial S} = (1 + f(t, T_1) * T_1) \frac{\partial F}{\partial Fut_S(T_1)} + \dots + (1 + f(t, T_N) * T_N) \frac{\partial F}{\partial Fut_S(T_N)}$$

$$\frac{\partial^2 G}{\partial S^2} = (1 + f(t, T_1) * T_1)^2 \frac{\partial^2 F}{\partial Fut_S(T_1)^2} + \dots + (1 + f(t, T_N) * T_N)^2 \frac{\partial^2 F}{\partial Fut_S(T_N)^2}$$

$$\frac{\partial G}{\partial f(T_i)} = T_i S \frac{\partial F}{\partial Fut_S(T_i)}$$

$$\frac{\partial^2 G}{\partial f(T_i)^2} = (T_i S)^2 \frac{\partial^2 F}{\partial Fut_S(T_i)^2}$$

$$\frac{\partial G}{\partial Basis(T_i)} = \frac{\partial F}{\partial Fut_S(T_i)}$$

$$\frac{\partial^2 G}{\partial Basis(T_i)^2} = \frac{\partial^2 F}{\partial Fut_S(T_i)^2}$$

The time-to-maturity T , defined in equation (1), has an Act / 360 day-count convention and is the difference between the spot settlement ($t+2$) and the contract maturity.

Thus, if today is December 5th the 1M contract maturity date would be January 5th (rolled to the next business day to January 6th), giving an offset of 32 days which would then be used at the T_i .

The implementation will try to include this day-count as accurately as possible.

Because the basis risk factors are a function of the futures contracts, we will need to add these new risk factors corresponding to sensitivities that we currently get for each futures contract.

The proposed basis risk factors for each precious metal are:

	XAG	XAU
1M	XAG_BASIS.M1M.CO.MKT.BASIS.PR.ABS	XAU_BASIS.M1M.CO.MKT.BASIS.PR.ABS
2M	XAG_BASIS.M2M.CO.MKT.BASIS.PR.ABS	XAU_BASIS.M2M.CO.MKT.BASIS.PR.ABS
3M	XAG_BASIS.M3M.CO.MKT.BASIS.PR.ABS	XAU_BASIS.M3M.CO.MKT.BASIS.PR.ABS
6M	XAG_BASIS.M6M.CO.MKT.BASIS.PR.ABS	XAU_BASIS.M6M.CO.MKT.BASIS.PR.ABS
12M	XAG_BASIS.M12M.CO.MKT.BASIS.PR.ABS	XAU_BASIS.M12M.CO.MKT.BASIS.PR.ABS
18M	XAG_BASIS.M18M.CO.MKT.BASIS.PR.ABS	XAU_BASIS.M18M.CO.MKT.BASIS.PR.ABS

	XPD	XPT
1M	XPD_BASIS.M1M.CO.MKT.BASIS.PR.ABS	XPT_BASIS.M1M.CO.MKT.BASIS.PR.ABS
2M	XPD_BASIS.M2M.CO.MKT.BASIS.PR.ABS	XPT_BASIS.M2M.CO.MKT.BASIS.PR.ABS
3M	XPD_BASIS.M3M.CO.MKT.BASIS.PR.ABS	XPT_BASIS.M3M.CO.MKT.BASIS.PR.ABS
6M	XPD_BASIS.M6M.CO.MKT.BASIS.PR.ABS	XPT_BASIS.M6M.CO.MKT.BASIS.PR.ABS
		XPT_BASIS.M12M.CO.MKT.BASIS.PR.ABS
		XPT_BASIS.M18M.CO.MKT.BASIS.PR.ABS

Reference:

<https://finpricing.com/lib/IrCurve.html>