Futures Convexity Adjustment

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From Eurodollar futures prices, we get implied futures rates. However, these rates can not be used directly as forward rates, because futures are priced on the spot base. An adjustment is required (called convexity adjustment) to modify a futures rate to get the corresponding forward rate.

As discussed in the Hull-White model, the convexity adjustments can be calculated in the Hull-White model framework.

We implemented the Hull-White model approach when the mean reversion factor was set to zero (the Ho-Lee model). The results match what we obtained from brokers. Here is the example on January 3, 2008:

FUTURES	**Price**	**LIBOR Vol**	**Convxty Adj**	**Broker**	**Difference**
19-Mar-2008	95.9025	24.531%	0.0005%	0.0020%	**-0.0015%**
18-Jun-2008	96.4025	29.150%	0.0018%	0.0030%	**-0.0012%**
17-Sep-2008	96.6525	32.426%	0.0040%	0.0050%	**-0.0010%**
17-Dec-2008	96.7625	34.840%	0.0074%	0.0080%	**-0.0006%**
18-Mar-2009	96.7675	35.173%	0.0115%	0.0120%	**-0.0005%**
17-Jun-2009	96.6675	35.234%	0.0173%	0.0180%	**-0.0007%**
16-Sep-2009	96.5250	32.953%	0.0220%	0.0240%	**-0.0020%**

16-Dec-2009	96.3600	30.415%	0.0265%	0.0320%	**-0.0055%**
17-Mar-2010	96.1925	29.126%	0.0333%	0.0400%	**-0.0067%**
16-Jun-2010	96.0125	27.940%	0.0411%	0.0480%	**-0.0069%**
15-Sep-2010	95.8650	26.755%	0.0487%	0.0560%	**-0.0073%**
15-Dec-2010	95.7425	25.569%	0.0559%	0.0640%	**-0.0081%**
16-Mar-2011	95.6550	24.946%	0.0650%	0.0720%	**-0.0070%**
15-Jun-2011	95.5650	24.342%	0.0744%	0.0790%	**-0.0046%**
21-Sep-2011	95.4825	23.691%	0.0846%	0.0870%	**-0.0024%**
21-Dec-2011	95.3950	23.089%	0.0949%	0.0950%	**-0.0001%**
21-Mar-2012	95.3300	22.513%	0.1045%	0.1020%	**0.0025%**
20-Jun-2012	95.3125	21.937%	0.1150%	0.1110%	**0.0040%**
19-Sep-2012	95.2350	21.360%	0.1217%	0.1200%	**0.0017%**
19-Dec-2012	95.1550	20.832%	0.1322%	0.1300%	**0.0022%**

The convexity adjustment is given as

$$\frac{\sigma^2}{2} t_1 t_2$$

where:

σ: local volatility

t_1 : Futures starting time

t_2 : Futures ending time

To get local volatility in the Ho-Lee model, we follow the following steps:

1. Let F be the futures price, and let σ_L be the LIBOR volatility. The futures rate is $1-F$. The corresponding bond price is approximately

$$P = \frac{1}{1+(t_2-t_1)(1-F)}$$

Then the volatility σ_P of P is

$$\sigma_P = (t_2-t_1)\sigma_L P(1-F)$$

In the Ho-Lee model, the volatility of a bond price is $\sigma(t_2-t_1)$. Therefore, the local volatility is

$$\begin{aligned}\sigma &= \sigma_L P(1-F) \\ &= \sigma_L \frac{1}{1+(t_2-t_1)(1-F)}(1-F)\end{aligned}$$

Reference:

https://finpricing.com/lib/IrCurveIntroduction.html