Callable Bond and Vaulation
Callable Bond

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Callable Bond

Callable Bond Definition

◆ A callable bond is a bond in which the issuer has the right to call the bond at specified times (callable dates) from the investor for a specified price (call price).

◆ At each callable date prior to the bond maturity, the issuer may recall the bond from its investor by returning the investor’s money.

◆ The underlying bond can be a fixed rate bond or a floating rate bond.

◆ A callable bond can therefore be considered a vanilla underlying bond with an embedded Bermudan style option.

◆ Callable bonds protect issuers. Therefore, a callable bond normally pays the investor a higher coupon than a non-callable bond.
Callable bond

Advantages of Callable Bond

- Although a callable bond is a higher cost to the issuer and an uncertainty to the investor comparing to a regular bond, it is actually quite attractive to both issuers and investors.
- For issuers, callable bonds allow them to reduce interest costs at a future date should rate decrease.
- For investors, callable bonds allow them to earn a higher interest rate of return until the bonds are called off.
- If interest rates have declined since the issuer first issues the bond, the issuer is likely to call its current bond and reissues it at a lower coupon.
Callable Bond

Callable Bond Payoffs

◆ At the bond maturity $T$, the payoff of a callable bond is given by

$$V_c(t) = \begin{cases} 
F + C & \text{if not called} \\
\min(P_c, F + C) & \text{if called}
\end{cases}$$

where $F$ – the principal or face value; $C$ – the coupon; $P_c$ – the call price; $\min(x, y)$ – the minimum of $x$ and $y$

◆ The payoff of the callable bond at any call date $T_i$ can be expressed as

$$V_c(T_i) = \begin{cases} 
\overline{V}_{T_i} & \text{if not called} \\
\min(P_c, \overline{V}_{T_i}) & \text{if called}
\end{cases}$$

where $\overline{V}_{T_i}$ – continuation value at $T_i$
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Model Selection Criteria

◆ Given the valuation complexity of callable bonds, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numerical solution to price them numerically.

◆ The selection of interest rate term structure models

  ◆ Popular interest rate term structure models:
    Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).

  ◆ HJM and LMM are too complex.

  ◆ Hull-White is inaccurate for computing sensitivities.

  ◆ Therefore, we choose either LGM or QGM.
The selection of numeric approaches

- After selecting a term structure model, we need to choose a numerical approach to approximate the underlying stochastic process of the model.
- Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.
- Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.
- Therefore, we choose either PDE or lattice.

- Our decision is to use LGM plus lattice.
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LGM Model

- The dynamics
  \[ dX(t) = \alpha(t)dW \]
  where \( X \) is the single state variable and \( W \) is the Wiener process.
- The numeraire is given by
  \[ N(t, X) = \frac{H(t)X + 0.5H^2(t)\zeta(t)}{D(t)} \]
- The zero coupon bond price is
  \[ B(t, X; T) = D(T)exp\left(-H(t)X - 0.5H^2(t)\zeta(t)\right) \]
The LGM model is mathematically equivalent to the Hull-White model but offers:
- Significant improvement of stability and accuracy for calibration.
- Significant improvement of stability and accuracy for sensitivity calculation.
- The state variable is normally distributed under the appropriate measure.
- The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfectly correlated.
Callable Bond

LGM calibration

- Match today’s curve
  
  At time $t=0$, $X(0)=0$ and $H(0)=0$. Thus $Z(0,0;T)=D(T)$. In other words, the LGM automatically fits today’s discount curve.

- Select a group of market swaptions.

- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.
Callable Bond

Valuation Implementation

- Calibrate the LGM model.
- Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- Calculate the payoff of the callable bond at each final note.
- Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date.
- Compare exercise values with intrinsic values at each exercise date.
- The value at the valuation date is the price of the callable bond.
Callabe Bond

A real world example

<table>
<thead>
<tr>
<th>Bond specification</th>
<th>Callable schedule</th>
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<tbody>
<tr>
<td><strong>Buy Sell</strong></td>
<td><strong>Buy</strong></td>
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<td>Calendar</td>
<td>Call Price</td>
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<td>Interest Accrual Date</td>
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<tr>
<td>Issue Date</td>
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<td><strong>Coupon</strong></td>
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Thanks!

You can find more details at https://finpricing.com/faq.html