Dividend Risk Modeling and Methodology

In practice, most traders treated dividend amount as "known" and risk managers do not monitor the price sensitivity to dividend amount as long as the total price sensitivity to underlying asset price is properly captured in case the dividend yield is used in pricing. If they think there is any dividend risk, that is only associated with the big negative jumps described above. Stress test, rather than daily VaR, is a proper way to capture that kind of risk.

We should bear in mind that the above view is only based on generic equity and equity derivative portfolios. As we can see, some derivative, such as long dated equity forwards and options, as well as dividend swaps, are very sensitive to expected dividend amount. Clearly, for these positions the dividend risk is no longer immaterial.

At any point of time, the dividend yield of a dividend-paying stock can be written as

$$y_T(t) = \frac{\frac{1}{T-t} E_t^Q \left[\sum_{t<\tau_i \le T} D(\tau_i) \right]}{S(t)} \tag{1.1}$$

Where

$y_T(t)$: dividend yield for term T-t at valuation time t

$S(t)$: stock price at valuation time t

$D(\tau_i)$: dividend amount at future time τ_i, *taken as present value*

$E_t^{\,Q}[\cdot]$: expectation under the risk-neutral measure at time t

The above gives a term structure of dividend yield in the same way that interest rate term structure (see https://finpricing.com/lib/IrCurve.html) is derived. For simplicity, we assume that percentage changes of dividend expectations at different holding periods are driven by the same risk factor and of same magnitude. Hence, using one-year constant holding period, the yield in (1.1) is short-handed as y_t.

As we emphasized above (i.e. implied versus realized), the forward dividends instead of trailing dividends are used in yield calculation.

Note that in financial mathematical term, the *continuous* dividend yield is defined as in the following equation:

$$D_t = S(t)\cdot(1-e^{-y_T(t)\cdot(T-t)}) \cong S(t)\cdot y_T(t)\cdot(T-t) \quad as\ y_T(t)\cdot(T-t) << 1 \qquad (1.2)$$

Where

$$D_t = E_t^{\,Q}\left[\sum_{t<\tau_i\le T} D(\tau_i)\right]$$

We observe that (1.2) is equivalent to (1.1) when yield is very small and holding time is short. The index dividend yield is typically 2% to 3%. The consistency of (1.2) and (1.1) harmonizes the market definition of dividend yield with the continuous dividend yield to be used as a pricing input of equity derivative instruments.

Unlike bond coupon, the perceived dividend is not guaranteed. The market expectation of present value of total dividend to be received in a constant holding period may change day to day. This is due to the same reasons that drive the fluctuation of the market stock price. We assume that the *innovation* at time t+1,denoted as ξ_{t+1}, is exogenous:

$$D_{t+1} = (1+\xi_{t+1})\cdot D_t \qquad (1.5)$$

Also, we assume one day return on stock price (dividend-adjusted) at time t is μ_t

$$\mu_t = \frac{S_t - S_{t-1}}{S_{t-1}} \qquad (1.6)$$

From (1.1), (1.5) and (1.6) we can derive

$$\frac{y_{t+1}}{y_t} = \frac{1+\xi_{t+1}}{1+\mu_{t+1}} \text{ or}$$

$$\frac{\Delta y_{t+1}}{y_t} = \frac{\xi_{t+1} - \mu_{t+1}}{1+\mu_{t+1}} \text{ where } \Delta y_{t+1} = y_{t+1} - y_t \qquad (1.7)$$

From (1.7) the relative change of dividend yield is calculated from the stock return and relative change of the expected dividend amount. For the latter we need to introduce a new risk factor ξ.

This new risk factor ξ, interpreted as the daily relative change of expectation on dividend amount, is due to uncertainty with regard to the actual dividends to be received by shareholder. Even with the most stable dividend paying company, strictly speaking, the uncertainty still exist, albeit small. Two reasons cause uncertainty: first, a company only announces the next quarterly dividend without committing itself to the quarters beyond; secondly, even with announced dividend, there are still chances that dividend can be sliced (e.g. GE in early 2009) or eliminated entirely (Citigroup in 2008). Market expectation will be updated constantly based on the news and market general sentiment. With this in mind, we need to model daily innovation in dividend expectation, ξ, as a random variable.

In Appendix C, we investigate the day-to-day changes of the expected dividend amount of stocks and equity indices. We observe that

- the distribution of daily change of expected dividend amount is predominantly concentrated at zero
- the variability of daily change of expected dividend amount is only a fraction of the variability of daily change of stock price (i.e., daily stock return)

We will discuss the dividend risk model in the context of single stock and basket of stocks (including equity index) separately. The behavior of the daily innovation in dividend expectation, ξ, for single stock and basket of stocks are apparently different based on empirical observation and hence warrant different models.

This category includes equity index ETF.

Empirical data indicates that the dividend expectation may not change every day. Innovation is mostly event driven for single stock and hence appears to be a jump process. The jump frequency and jump size can be estimated from the expected dividend amount time series.

Without loss of generality, we assume that the daily innovation (relative change) in dividend expectation ξ has the mixed distribution, described as follows

$$\xi = \begin{cases} 0 & \text{with probability } q \\ \tilde{\xi} & \text{with probability } p \end{cases}, \text{ where } q = 1 - p \text{, and} \tag{1.8}$$

$$\tilde{\xi} \sim double\exp onential(\lambda)$$

Pricing of equity derivatives on single stock usually takes expected dividend amount as input. Sensitivity with respect to expected dividend amount is readily available. Directly modeling changes to dividend expectation is a very convenient way to capture risk via DGV approach.

In case that the dividend yield is used in pricing derivative on single stock, sensitivity is calculated with respect to dividend yield. The changes to dividend yield can be derived from changes to dividend expectation using (1.7). Then the dividend risk can be captured via DGV approach.

For most stable dividend-paying stocks such as the banks and utility companies, market expectation is most often based on the constant extrapolation of the announced next quarterly dividend. The day-to-day changes to the expectation of the dividend amount to be paid by these companies are typically quite small. In practice, traders tend to ignore the slim uncertainty about dividend of these stocks and instead take the dividend as known. In this case, it may not be too wrong to assume that the dividend amount is known. That is equivalent to $\xi_{t+1} = 0$ in the more general case in (1.5) and $p = 0$ in (1.8).

Pricing of equity derivative on basket of stocks or equity index requires dividend yield as input. Sensitivity with respect to dividend yield is either readily available or can be calculated easily.

We assume that the daily innovation (relative change) in dividend expectation ξ has the double exponential distribution

$$\xi \sim double\,\exp onential(\lambda) \qquad (1.10)$$

The disappearance of jumps, as in the case of changes of expected dividend of single stock, is due to two reasons: First, the size of the innovation usually is more moderate due to diversification; Secondly, the frequency of occurrence of events that impact the expected amount of aggregated dividend is much higher than in the case of single stock. Empirical data supports this assumption.