Commodity Volatility Closing Rate Calculation

We use the Omega type models to model the implied volatility skew for commodity derivatives. These surfaces are incorporate moneyness and time to maturity into the skew definition and are a function of calibrated parameters (skew, wing, boost etc).

To populate the closing rate for implied volatility skew risk factors, it will be necessary to implement the forward-delta based Omega skew models and use the Brent root finding algorithm to solve for the closing rate. This function spec will outline this procedure.

The inputs into the Omega model are the time-to-maturity, *tmat*, moneyness, $x=F/K$ where F is the price of the futures contract and K the strike. Other inputs are the at-the-money volatility, σ_{ATM}, skew, S, wing, W, and boost, B.

Thus we have $\sigma_\Omega(tmat, x, \sigma_{ATM}, S, W, B)$ where:

$\sigma = \sigma_\Omega(tmat, x, \sigma_{ATM}, S, W, B)$

tmin=1e-7

t=max(tmin,tmat)

vt=((σ_{ATM} + B)/100)*sqrt(t)

Δ_{ATM} = normcdf(vt/2)

$\Delta_{k.}$ = normcdf(ln(x)/vt+vt/2)

$\Omega = 2*(\Delta_{ATM} - \Delta_k)$

$\sigma = \sigma_{ATM} + S * \Omega + W * \Omega^2$

The inputs for the OmegaQ model are the same as above plus the parameters: cube, *C*, and quart, *Q*.

Thus we have $\sigma_{\Omega Q}(tmat, x, \sigma_{ATM}, S, W, B, C, Q)$ where:

$\sigma = \sigma_{\Omega Q}(tmat, x, \sigma_{ATM}, S, W, B, C, Q)$

tmin=1e-7

t=max(tmin,tmat)

vt=((σ_{ATM} + B)/100)*sqrt(t)

Δ_{ATM} = normcdf(vt/2)

$\Delta_{k.}$ = normcdf(ln(x)/vt+vt/2)

$\Omega = 2*(\Delta_{ATM} - \Delta_k)$

$\sigma = \sigma_{ATM} + S * \Omega + W * \Omega^2 + C * \Omega^3 + Q * \Omega^4$

The OmegaX skew model is comprised of two quasi-independent Omega skews. From the data feed the parameters are once again the time-to-maturity, *tmat*, moneyness, *x*, and *Z1*, *Z2*, *d1*, *d2*, *d3*, *d4*, *l*, *h*, and *B*.

Thus we have $\sigma_{\Omega X}(tmat, x, \sigma_{ATM}, Z1, Z2, d1, d2, d3, d4, l, h, B)$ where:

$\sigma = \sigma_{\Omega X}(tmat, x, \sigma_{ATM}, Z_1, Z_2, d_1, d_2, d_3, d_4, l, h, B)$

tmin=1e-7

t=max(tmin,tmat)

vt=((σ_{ATM} + B)/100)*sqrt(t)

Z=ln(x)/vt

l=l+0.000001

A=1-2*normcdf(-Z1/100)

B=1-2*normcdf(-Z2/100)

denom=A*B*(B-A)+0.000001

$a_1=(d_1*A^2-d_2*B^2)/denom$

$a_2=(d_1*A-d_2*B)/denom$

$b_1=-1*(d_4*A^2-d_3*B^2)/denom$

$b_2=(d_4*A-d_3*B)/denom$

$c_0=normcdf((Z_2+h)/l)$

d_0=normcdf((Z_1+h)/l)

e_0=normcdf((h-Z_1)/l)

f_0=normcdf((h-Z_2)/l)

f_1=-(e_0*A+f_0*B)/(e_0*A^2+f0*B^2)

g_1=((1-c_0)*B+(1-d_0)*A)/((1-c_0)*B^2+(1-d_0)*A^2)

q_1=f_1*(c_0*B^2+d_0*A_2)-(c_0*B+d_0*A)

r_1=((1-e_0)*A+(1-f_0)*B)+g_1*((1-e_0)*A^2+(1-f_0)*B^2)

zz_1=d_1/B-(B*b_2-b_1)

zz_2=d_2/A-(A*b_2-b_1)

zz_3=d_3/A-(A*a_2+a_1)

zz_4=d_4/B-(B*a_2+a_1)

ε=((1-c_0)*B*zz_1+(1-d_0)*A*zz_2)/q_1

Δ=(e_0*A*zz_3+f_0*B*zz_4)/r_1;

a_1=a_1+ ε

a_2=a_2+ ε *f_1

b_1=b_1+ Δ

b_2=b_2+ Δ*g_1

Ω=1-2*normcdf(Z)

n=normcdf((Z+h)/l)

f=n*(a_1*Ω +a_2*$Ω^2$)+(1-n)*(b_1* Ω +b_2* $Ω^2$)

$\sigma = \sigma_{ATM} + f$

We now outline the process for populating the closing rates into system. As is outlined in the MRA Functional Specifications, we have the deal specific volatility $\sigma=\sigma(tmat,x,\beta)$ where r is the interest rate interpolated at *tmat*, x is the moneyness, and β is the set of parameters corresponding to the respective Omega model. Also, we have the forward delta based on the skew surface (σ_i,η_i) corresponding to the forward delta $\eta_i=\eta(tmat,x,\sigma_i)$. Thus, there are two equations and two unknown variables, x and σ_i.

To solve the equation, we make the assumption that η_i has the form of the Black-Scholes delta. Denoting $\eta_{C,i}$ and $\eta_{P,i}$ the delta for the call and put option, we see that that the delta's are:

$$\eta_{c,i} = \Phi\left(\frac{\ln(x)}{\sigma_i(tmat,x,\beta)\cdot\sqrt{tmat}} + 0.5\cdot\sigma_i(tmat,x,\beta)\cdot\sqrt{tmat}\right)$$

$$\eta_{p,i} = -\Phi\left(-\frac{\ln(x)}{\sigma_i(tmat,x,\beta)\cdot\sqrt{tmat}} - 0.5\cdot\sigma_i(tmat,x,\beta)\cdot\sqrt{tmat}\right)$$

Using the above definitions of delta, we want to solve for the moneyness given $\eta_{C,i}$ and $\eta_{P,i}$. The equations we need to solve are below stated below.

$$0 = \frac{\ln(x)}{\sigma_i(tmat,x,\beta)\cdot\sqrt{tmat}} + 0.5\cdot\sigma_i(tmat,x,\beta)\cdot\sqrt{tmat} - \Phi^{-1}(\eta_{c,i})$$

For the put options:

$$0=\frac{\ln(x)}{\sigma_i(tmat,x,\beta)\cdot\sqrt{tmat}}+0.5\cdot\sigma_i(tmat,x,\beta)\cdot\sqrt{tmat}+\Phi^{-1}(\eta_{p,i})$$

Given $\eta_{P,10D} = 0.10$, $\eta_{P,25D} = 0.25$, $\eta_{C,25D} = 0.25$, and $\eta_{C,10D} = 0.10$, we solve for the moneyness: x^{10DP}, x^{25DP}, x^{25DC}, x^{10DC}. Following this, we plug the moneyness back into the corresponding Omega model to get the closing rate:

$\sigma^{10DP} = \sigma(tmat, x^{10DP},\beta)$

$\sigma^{25DP} = \sigma(tmat, x^{25DP},\beta)$

$\sigma^{25DC} = \sigma(tmat, x^{25DC},\beta)$

$\sigma^{10DC} = \sigma(tmat, x^{10DC},\beta)$

The suggested method for solving the moneyness is the Brent root finding method. This will be outlined in the Appendix.

The vega allocation is outlined in the MRA functional spec on the skew introduction into system. Here we briefly review the methodology and then go through an example of vega allocation.

Assume that the implied volatility $\sigma(t,T,m)$ is the function of option term T and moneyness m (which in this case is the forward delta). Also assume that risk factors are chosen at fixed points $(T_i, m_j), 0 \le i \le n, 0 \le j \le l$, since that the vegas from the source system may not coincide with these standard points, suitable allocation of the vegas is necessary.

Assume the vega $v(T,m)$ at (T,m),

1. If $T_{i-1} < T \le T_i, m_{j-1} < m \le m_j$, then $v(T,m)$ will be allocated over four points: $(T_{i-1}, m_{j-1}), (T_{i-1}, m_j), (T_i, m_{j-1}), (T_i, m_j)$ respectively as

$$\begin{array}{l} v(T,m)(1-u)(1-v), \\ v(T,m)(1-u)v, \\ v(T,m)u(1-v), \\ v(T,m)uv \end{array} \quad \text{, with}$$

$$u = \frac{T - T_{i-1}}{T_i - T_{i-1}}, v = \frac{m - m_{j-1}}{m_j - m_{j-1}}$$

2. If T_0 is the left end point and $T \le T_0$, then $u = 0$; allocation only happens among (T_0, m_{j-1}) and (T_0, m_j);
3. If T_n is the right end point and $T \ge T_n$, then $u = 1$; allocation only happens among (T_n, m_{j-1}) and (T_n, m_j);
4. Similarly, if m_0 is the left end point and $m \le m_0$, then $v = 0$; allocation only happens among (T_{i-1}, m_0) and (T_i, m_0);
5. If m_l is the right end point and $m \ge m_l$, then $v = 1$; allocation only happens among (T_{i-1}, m_l) and (T_i, m_l);
6. If $T \le T_0$, $m \le m_0$, then $u = 0$ and $v = 0$; allocation only happens at (T_0, m_0);

One way to think of the u and the v above, are as the weights used in linear interpolation. We now look at the vega allocation for a specific deal, 25878, an Asian call option on NYM_WTI. Here is relevant information for the deal specific to vega allocation:

Reference:

https://finpricing.com/lib/IrCurveIntroduction.html